## **Trial Higher School Certificate Examination**

2002



# **Mathematics**

#### **General Instructions**

- Reading time 5 minutes
- Working time 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

#### Total marks (120)

- Attempt Questions 1-10
- All questions are of equal value

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

## Question 1 (12 marks)

a) Evaluate 
$$\frac{23.97 - (3.62)^2}{\sqrt{4.51}}$$
 correct to 2 decimal places

1

b) Solve: 
$$\frac{x}{3} - \frac{2x+1}{4} = 5$$

2

c) Sketch 
$$y = \sin 2x$$
 for  $0 \le x \le 2\pi$ 

2

d) Solve: 
$$|1-2x| < 4$$

3

e) Evaluate 
$$\log_e 3.5$$
, correct to 2 decimal places

1

f) If 
$$\frac{dy}{dx} = x^2 + 8x$$
 and when  $x = 3$ ,  $y = 0$  find y in terms of x

#### Question 2 (12 marks)

a) Differentiate:

(i) 
$$y = (2x-5)^4$$

2

(ii) 
$$y = e^{2x} \sin x$$

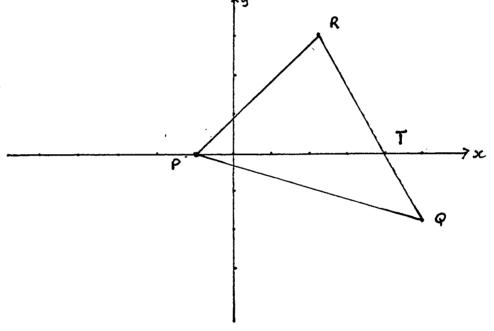
2

b) Using the information on the following diagram



R (2,3)

9 (5,-2)



(i) Show that the equation of the line 
$$PQ$$
 is  $x+3y+1=0$ 

2

(ii) Find the length of PQ

1

(iii) Find the perpendicular distance from R to PQ

2

(iv) Find the area of  $\Delta PRQ$ 

1

(v) Find the size of the angle RTP correct to nearest degree

## Question 3 (12 marks)

a) Prove  $\csc x \cos x \tan x = 1$ 

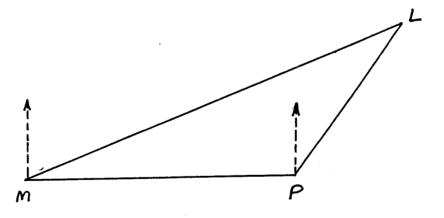
1

b) Given that  $\cot \beta = \frac{3}{2}$  and  $\sin \beta < 0$  find exact value of  $\cos \beta$ 

2

c) Solve  $2\sin^2 x + \cos x = 2$  for  $0^\circ \le x^\circ \le 360^\circ$ 

- 3
- d) The bearing of a lighthouse L from a ship at M is  $N55^{\circ}E$ . The ship then sails due East from M to a point P which is 10 nautical miles from L. The bearing of the lighthouse from P is  $N25^{\circ}E$ .



- (i) Copy the diagram into your answer booklet.
- (ii) Deduce that  $M\hat{L}P = 30^{\circ}$

1

- (iii) Show that  $MP = 5\csc 35^{\circ}$  and hence find MP correct to 1 decimal place.
- 3

2

(iv) If the ship continues to sail due East from P find its shortest distance from the lighthouse.

## Question 4 (12 marks)

- a) Find the primitive of:
  - (i)  $3xe^{x^2}$

2

(ii)  $\tan x$ 

2

b) (i) Sketch the curve  $y = \ln(4-x)$  showing the x and y intercepts

2

(ii) Find the equation of the tangent at the point where the curve crosses the x axis

2

(iii) Find the exact area bounded by the curve and the x and y axes

#### Question 5 (12 marks)

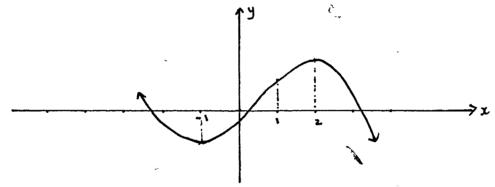
a) Using the following table of values find an approximation to the value of:

2

$$\int_{1}^{5} f(x)dx$$
 using Simpson's Rule with 5 function values

ſ	x	1	2	3	4	5
	f(x)	1.74	3.9	4.2	7.89	10.2

b) The following is a graph of some function y = f(x).



(i) In your answer booklet graph y = f'(x) and y = f''(x) if at x = 1 there is a point of inflexion.

1

3

(ii) Explain what happens to the curve at a point of inflexion.

4

c) (i) Determine the stationary points and their nature for the curve  $y = x^3 - 6x^2 + 9x - 7$ 

1

(ii) Sketch the curve

1

(iii) For what x values is the curve increasing?

2

1

3

1

#### Question 6 (12 marks)

remains?

- Zinc is extracted from a mine at a rate that is proportional to the amount of zinc a) remaining in the mine. Hence the amount R remaining after t years is given by  $R = R_o e^{-kt}$  where k is a constant and  $R_o$  is the initial amount of zinc. After 5 years, 50% of the initial amount of zinc remains.
  - Find the value of k (correct to 4 decimal places) (i)
  - (ii) How many more years will elapse before only 30% of the original amount 3
- Joanne decided to save for an overseas holiday. She decided to deposit \$500 into a special account at the beginning of each month for 3 years. The account paid 6% pa compounded monthly.
  - How much is the first payment of \$500 worth at the end of 3 years? (i)
  - (ii) Prove, by developing a geometric series, that the total value of all her deposits at the end of 3 years is given by
    - Total value =  $100\ 500\ (1.005^{36}-1)$
  - (iii) Calculate this value.
  - (iv) If Joanne had needed a lump sum of \$22 000 by the end of the third year what would she have had to save each month? (You may use any previous working). 2

## Question 7 (12 marks)

a) The limiting sum of the infinite geometric series  $1+5^x+5^{2x}+...$  is 5. Find x (correct to 3 decimal places).

3

- b) Find the x values of the intersection points of the curves x-y=4 and  $y=x^2-3x-4$ . Hence, find the area between the two curves.
- 4

c) (i) Write down the discriminant of:  $3x^2 + 2x + k$ .

7

- (ii) For what values of k does  $3x^2 + 2x + k = 0$  have real roots?
- d) If one root of the equation  $mx^2 px + 1 = 0$  is double the other, prove that  $2p^2 = 9m$ .

#### Question 8 (12 marks)

- a) The velocity of a particle moving in a straight line is given by v = 2t 6 where position is measured in metres and time in seconds. If the particle is initially 4 metres to the right of the origin
  - (i) Find an expression for displacement.

1

(ii) Find when and where the particle comes to rest.

2

(iii) Find the total distance covered by the particle in the first 5 seconds.

1

(iv) Evaluate  $\int_0^5 (2t-6) dt$ 

2

(v) Explain why your answers for parts (iii) and (iv) are not the same.

1

- b) The rate at which gas escapes from a balloon is given by  $\frac{dG}{dt} = \frac{-3}{t+1}$  where the gas is measured in cm<sup>3</sup> and time in seconds.
  - (i) Find G as a function of time if the initial amount of gas in the balloon is  $10 \text{cm}^3$ .

2

(ii) How long before all the gas has escaped?

## Question 9 (12 marks)

a) Find:

(i) 
$$\int \sin^2 x \cos x \, dx$$

1

(ii) 
$$\int_0^{\frac{\pi}{4}} (2\sin x - \cos 2x) \ dx$$

3

b) Sketch the curve  $y = 2\cos \pi x$  for  $0 \le x \le 2$ 

1

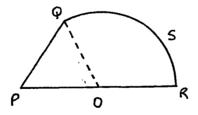
On the same set of axes sketch the graph y = 1 - x

1

Using your graph state how many solutions  $2\cos \pi x = 1 - x$  has in the domain  $0 \le x \le 2$ 

1

c) The region QPORS is formed by an equilateral triangle OPQ with a side of 12cm and a sector QORS. PR is a straight line. QSR is an arc of the circle centre O.



Giving answers in exact form find:

(i) The perimeter of the region

-2-

(ii) The area of the region

## Question 10 (12 marks)

a) (i) State domain and range of the function  $y = \sqrt{9-x}$ 

2

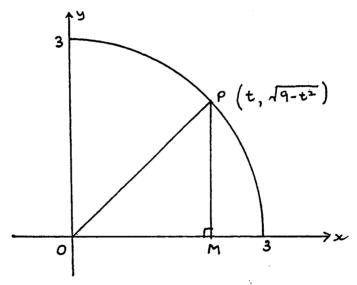
(ii) Sketch a graph of this function

1

(iii) Calculate the volume of the solid generated when the area bounded by the curve and the coordinate axes in the first quadrant is rotated about the y axis.

3

b) The diagram shows the curve  $y = \sqrt{9 - x^2}$  for  $x \ge 0$ . P is the point  $(t, \sqrt{9 - t^2})$  on the graph and M is the foot of the perpendicular from P to the x axis.



1

(i) Write down an expression in terms of t for the area A of the triangle OPM.

\_

(ii) Find the coordinates of the point P which gives triangle OPM a maximum area.

## MATHEMATICS - SOLUTIONS

#### QUESTION 1:

(a) 5.12 (cornect to 2 dec. pl)

(b) 
$$\frac{x}{3} - \frac{2x+1}{4} = 5$$

$$\frac{4x - 3(2x+1)}{12} = 5$$

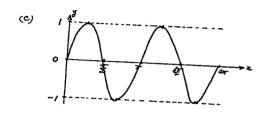
$$\frac{4x - 6x - 3}{12} = 5$$

$$\frac{-2x - 3}{12} = 5$$

$$-2x - 3 = 60$$

$$-2x = 63$$

$$\therefore x = -63$$



(d) 
$$|1-2x| < 4$$
  
 $\Rightarrow -4 < 1-2x < 4$   
 $-5 < -2x < 3$   
 $\frac{x}{4} > x > -\frac{3}{2}$   
 $\frac{1}{2} < x < \frac{x}{2}$ 

<u>.2.</u>

(f) 
$$\frac{dy}{dx} = x^2 + 8x$$
  
 $\Rightarrow y = \frac{x^3}{3} + 4x^2 + C$   
when  $x = 3, y = 0$   
 $\therefore 0 = 9 + 36 + C$   
 $\therefore C = -45$   
i.e.  $y = \frac{x^3}{3} + 4x^2 - 45$ 

<u>ુ.</u>

QUESTION 2:

(a) (i) 
$$\beta = (2x-5)^{4}$$
 $\frac{dy}{dx} = 4(2x-5)^{3}.2$ 
 $= 8(2x-5)^{3}$ 

(ii)  $\theta = e^{2x} \sin x$ 
 $\frac{dy}{dx} = Vn' + n V'$ 
 $= \sin x. 2e^{2x} + e^{2x} \cos x$ 
 $= e^{2x}(2 \sin x + \cos x)$ 

(b) (i) 
$$P(-1,0) = (5,-2)$$

$$m_{RR} = \frac{-2-0}{5--1}$$

$$= -\frac{1}{6}$$

$$=$$

(i) 
$$PQ = \sqrt{(5--1)^2 + (-2-0)^2}$$
  
=  $\sqrt{36 + 4}$   
=  $\sqrt{40}$   
=  $2\sqrt{10}$ 

4.

(iii) (2,3) to 
$$x + \frac{2}{3} + 1 = 0$$

$$d = \frac{1(2) + 3(3) + 1}{\sqrt{1^2 + 3^2}}$$

$$= \frac{12}{\sqrt{10}}$$

$$= \frac{12\sqrt{10}}{10}$$

$$= \frac{6\sqrt{10}}{5}$$
(iv) area of  $\Delta PRA = \frac{1}{2} \times PR \times d$ 

$$= \frac{1}{2} \times 2\sqrt{10} \times \frac{6\sqrt{10}}{3} \text{ (units)}$$

$$= 12(u-it^{2})$$

$$= 12(u-it^{2})$$

$$= -\frac{3}{2-5}$$

$$= -\frac{5}{5}$$

QUESTION 3:

(a) corecx core tour = 1 x tork x sine

(b) cot  $\beta = \frac{3}{2}$   $\Rightarrow \beta$  is in 3 th quadrant

 $2 \sqrt{13} \qquad \therefore \quad \cos \beta = -\frac{3}{\sqrt{13}}$ 

c)  $2 \sin^2 x + \cos x = 2$   $0 \le x^2 \le 260^2$   $2(1-\cos^2 x) + \cos x = 2$   $2-2\cos^2 x + \cos x = 0$ .  $\cos x(2\cos^2 x - \cos x) = 0$ 

.: Core = 0, ₹

(d)

: x = 60°, 90°, 270°, 200°

6.

10 Nantical Miles

(i) MLP = 180° - (35° + 90° + 25°) (angle sum S in 180°)

(ii) In DMLP by the dime Rule

MP

sindo = 10

sindo =

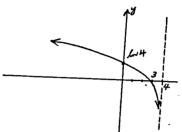
(iii) Using Cosine Rule  $LM^{2} = (8.717)^{2} + 10^{2} - 2 \times 8.717 \times 10 \cos 115^{\circ}$  = 249.66536

: LM = 15.80 nmiles (to 2 d.p.)

QUESTION 4:

(i)  $\int 3x e^{x^{2}} dx = \frac{3}{x} \int 2x e^{x^{2}} dx$   $= \frac{3}{x} e^{x^{2}} + c$ (ii)  $\int \tan x dx = \int \frac{\sin x}{\cos x} dx$   $= -\int \frac{-\sin x}{\cos x} dx$   $= -\log(\cos x) + c$ 

(b) (i) y = lu (4-x) Domain: 4-x>



x intercept at y=0

ie ln (4-x) = 0

4-x = 1

x = 3

y intercept at x=0

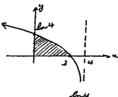
ie y = ln 4

(i)  $y = \ln(4-x)$   $\Rightarrow \frac{dy}{dx} = \frac{-1}{4-x}$ at (3,0)  $\frac{dy}{dx} = \frac{-1}{1}$  = -1  $\therefore \text{ Jangent at (3,0) in}$ 

2 2 + y - 3 = 0

(iii)  $A = \int_0^3 h (4-x) dx$ 

but as we are anable to find the primitive of lu (4-x) we must refer to area to the 7- aris as shown.



· A = \int z dy

 $g = \ln(4-x)$   $\Rightarrow e^{2} = 4-x$ ie  $x = 4-e^{2}$ 

= \int (4 - e) dy

= \left[4y - e^y]^{\text{en}4}

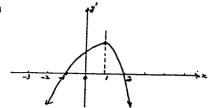
= \left(4 - e^{\text{en}4}\right) - \left(0 - 1)

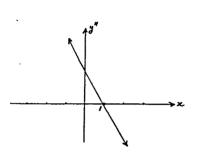
= 4 \left(4 - 4 + 1)

= 4 \left(4 - 3)

Area is (4 \left(4 - 3)) mix

(b) (i)





(ii) at a point of inflexion the curve changes cauity. Since the sign of f"(2) determines the concavity of a curve, we must have f"(x) = 0 at a point of inflexion AND the sign of f"(2) changing as we move along he curve from one side of the point to the other.

(c) (i) = 2 - 62 + 92 - 7 ---

 $2 = 3x^2 - 12x + 9$ 

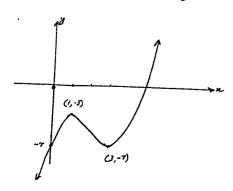
Stationary point at the -0

ie at (15-3) and (3,-7)

$$\frac{d^2y}{dx^2} = 6x - 12$$

at (1, 3) y" = -6 -> (1, -3) is a relative maxim turning point.

⇒ (3, -7) is a relative minimum tuming point



(iii) dressing for 2<1

 $R = R_0 e^{-AC}$ 

(i) at 
$$t=5$$
,  $R=\pm R$   

$$\pm R = Re^{-5h}$$

$$\Rightarrow e^{-5h} = \pm$$

.. another 3.7 years will elapse.

(b) (1) Trist \$500 grows to \$500 (1.005) = \$598.34

(ii) 1 st \$500 grows to \$500 (1.005) 2nd \$500 grows to \$500 (1.003) 5

last \$500 grows to \$500 (1.005)

.: Lump sum value

= \$500 (1.00s) + \$500 (1.00s) + ... + \$500 (1.00s) to Econotic series a = 500 (1.005) + 21.005 a 20

$$= \frac{a(x^{n}-1)}{x-1}$$

$$= \frac{500(1.005)(1.005^{16}-1)}{1.005-1} dollar (7)$$

\$19 766.39

(ir) O becomes

$$R = \frac{22 \cos(1.005-1)}{1.005(1.005^{k}-1)}$$

= \$100 500 (1.005 =1)

\$556.50 (connect to rearest cents

#### JESTION T:

1+5x+52x+... Geometric series a=1, +=5x S - 2

$$x + 1 = \log_5 4$$

$$x = \log_5 4 - 1$$

$$\log_5 5$$

$$0 + 0: z = 4 + x - 3z - 4$$

$$0 = x - 4x$$

$$0 = x(x - 4)$$

$$0 = x - kx$$

$$0 = x(x - 4)$$

$$x = 0, 4$$

$$A = \int_{0}^{4} \{z - 4 - (z^{2} - 3z - 4)\}^{2} dx$$

$$= \int_{0}^{4} (4x - x^{2}) dx$$

$$= \left(3z^{2} - \frac{1}{3}z^{3}\right)_{0}^{4}$$

$$= \left(3z - \frac{64}{3}\right) = 0$$

- 32 4.00 : 32 mits

(c) (i) 
$$3z + 2z + k$$
  

$$\Delta = k - 4ac$$

$$= 2 - 4(3)(k)$$

$$= 4 - 12k$$

(d) Let roots be 
$$\alpha$$
,  $2\alpha$   $m\tilde{x} - px + 1 = 0$ 

$$2 \times (p)^2 = 4$$

$$2p^2 = 4$$

$$\frac{2p}{9n} = \pi$$

QUESTION 8:

(a) (i) 
$$v = 5x = 2t-6$$
  
 $\therefore k = t^{-}6t+C$   
at  $t = 0, x = 4$   
 $\therefore 4 = C$   
 $\therefore x = t^{-}6t+4$ 

in comes to rest after 3 seconds and particle is 5 m to the left of O.

(ii) at t=0 
$$x=4$$
 $5=3$   $x=-5$ 
 $5=9+4$ 

(iv) 
$$\int_{0}^{\infty} (2t-6) dt = [t^{2}-6t]^{\infty}$$
  
= (25-30) - 0

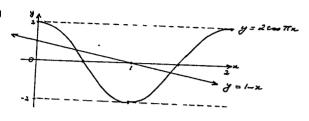
Jo (26-6) att is not the total distance travelled it is the change in displacement in the first 5 seconds. They would be the same if the particle did not stop and change directions in The first 5 seconds.

(b) (i)  $\frac{dG}{dt} = -\frac{3}{1+1}$ 

(ii) Gas escapes => G=0

: Time is 27 seconds (correct to m

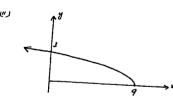
#### WESTION 9:



There are 2 solutions for 0 & 2 & 2.

#### QUESTION 10:

-: Domain is { 2: 2 5 9} Range in { 4: 4 > 0}



$$(iii) \qquad V = \pi \int_{0}^{3} x^{2} dy$$

$$V = \pi \int x^{2} dy$$

$$J = \sqrt{9-2}$$

$$X = 9-3^{2}$$

$$X = 81 - 183^{2} + 3^{4}$$

$$= \pi \int_{0}^{1} (81 - 183^{2} + 3^{4}) dy$$

$$= \pi \left[ 81y - 6y^{3} + \frac{3}{5} \right]_{0}^{1}$$

$$= \pi \left[ 243 - 162 + \frac{243}{5} - 0 \right]$$

$$= \frac{648\pi}{100}$$

.. Volume is 6487 units

19.

(b) (i) 
$$A = \frac{1}{2} \cdot besse \times \underline{I} \cdot besset$$

$$= \frac{1}{2} \cdot t \cdot \sqrt{9 \cdot t^2}$$

$$= \frac{1}{2} \cdot (9 - t^2)^{\frac{1}{2}}$$
(i)  $dA = (9 - t^2)^{\frac{1}{2}} \cdot t + t \cdot d \cdot (9 \cdot t^2)^{\frac{1}{2}}$ 

(i) 
$$\frac{dA}{dt} = (9-t^2)^{\frac{1}{2}} \frac{d}{dt} + \frac{t}{2} \frac{d}{dt} (9-t^2)^{-\frac{1}{2}} (-3t)$$

$$= \frac{\sqrt{9-t^2}}{2} - \frac{t^2}{2\sqrt{9-t^2}}$$

stationary point occurs when the a

in 
$$\frac{\sqrt{9-t^2}}{2}$$
 .  $\frac{t}{2\sqrt{9-t^2}}$  = 0

in  $\frac{\sqrt{9-t^2}}{2}$  =  $\frac{t}{2\sqrt{9-t^2}}$ 

$$2(9-t^2) = 2t^2$$

$$18-2t^2 = 2t^2$$

$$18 = 4t^2$$

$$t^2 = \frac{16}{4}$$

$$1 = \frac{3\sqrt{2}}{2} \text{ (since } t \ge 0)$$

TEST:

dA dt	2.1	3/2 2 0	2·2 -0·3	NOTE: hust give value, for this test !!

: havinum area occurs at  $t = \frac{31/2}{2}$ .: Co-ordinates of P are  $\left(\frac{31/2}{2}, \sqrt{9-\frac{9}{4}}\right)$   $= \left(\frac{31/2}{2}, \frac{31/2}{2}\right)$ 

3. 1